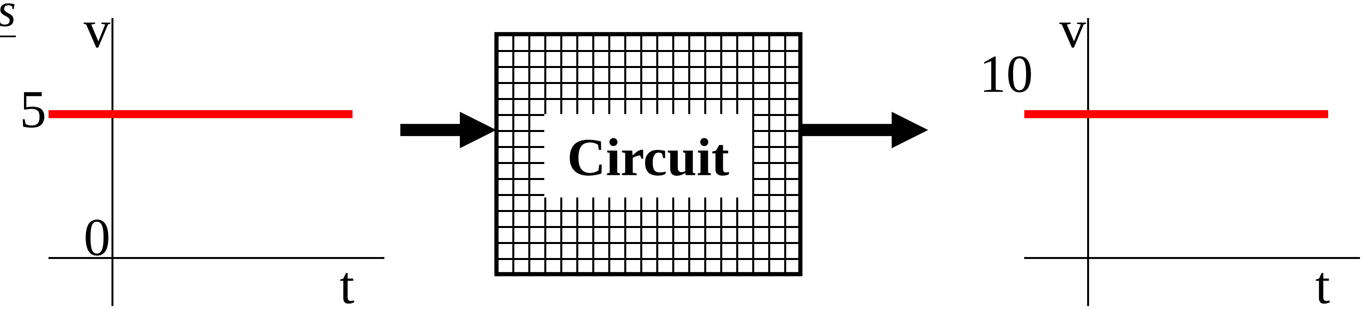
- DC Analysis
- AC Analysis
  (Sinusoidal Steady state Analysis)

*Different* **?**

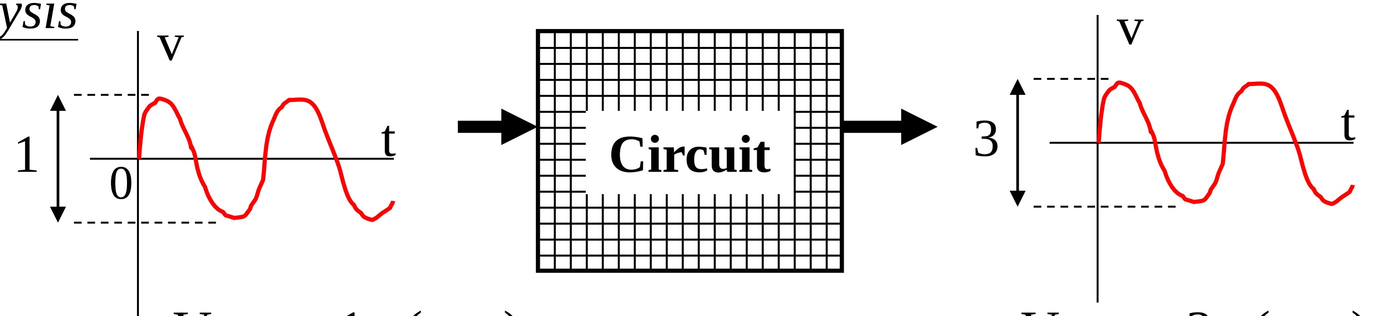
*DC analysis*




$V_{input}$ = 5v

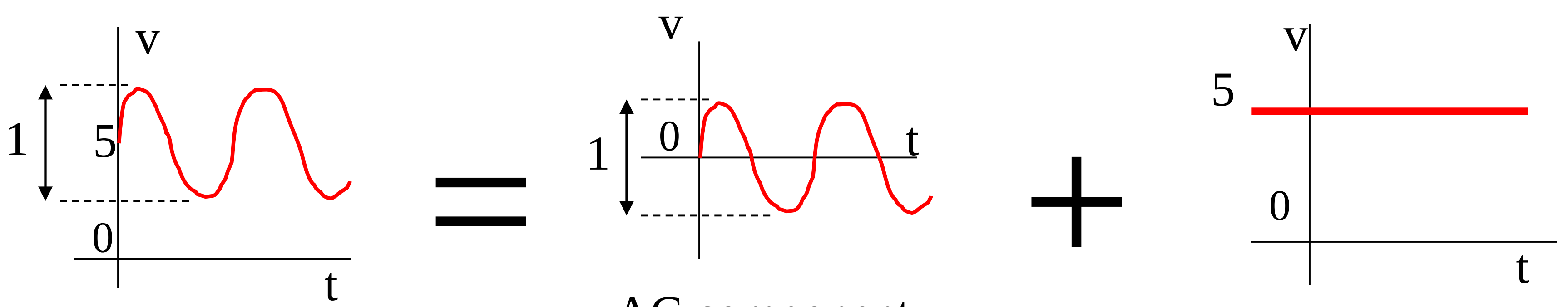
$V_{output}$ = 10v

*AC analysis*




$V_{input}$ = 1v (p.p.)

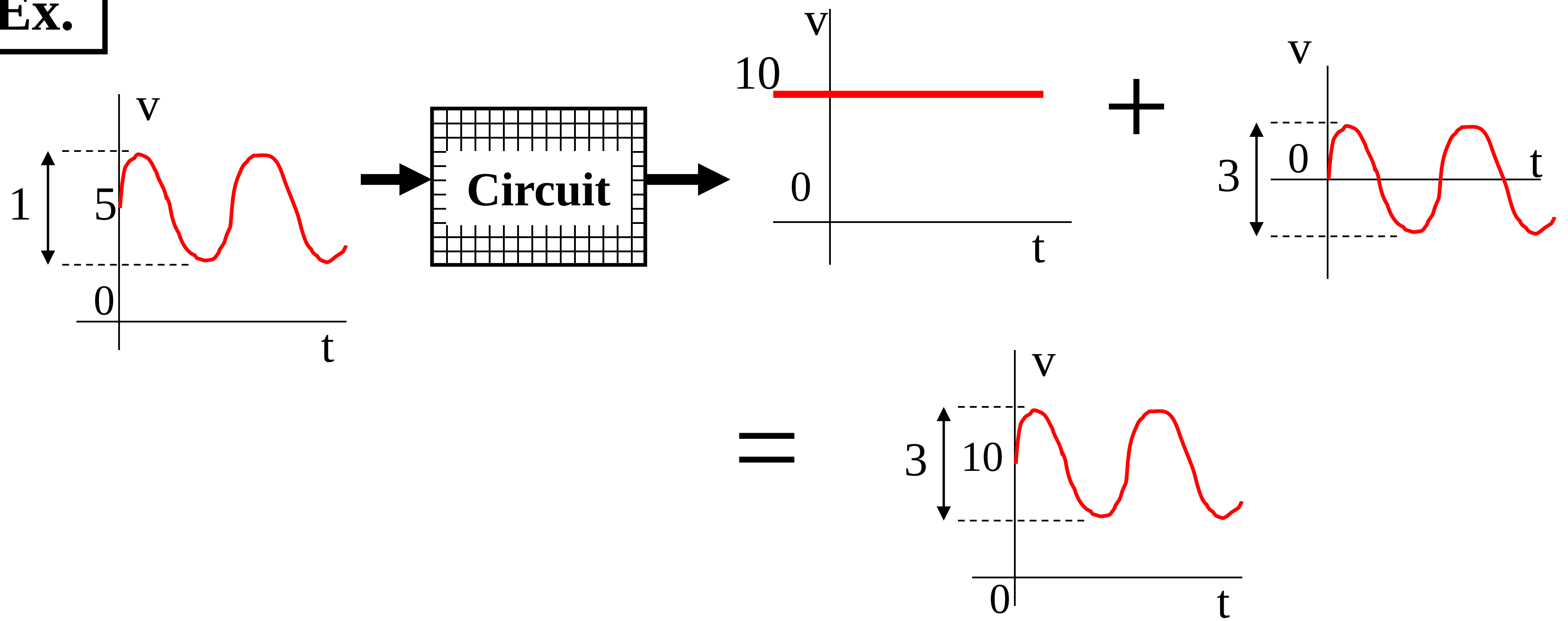
$V_{input}$ = 3v (p.p.)



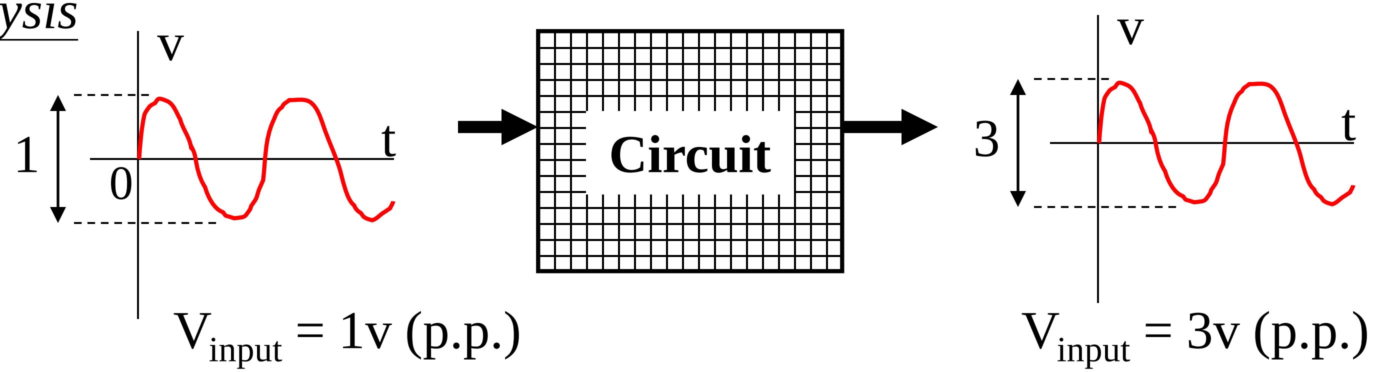

AC component

DC component

**Ex.**



*AC analysis*




## AC Analysis

Input Source

-- sin wave [e.g. sin(100t) ]

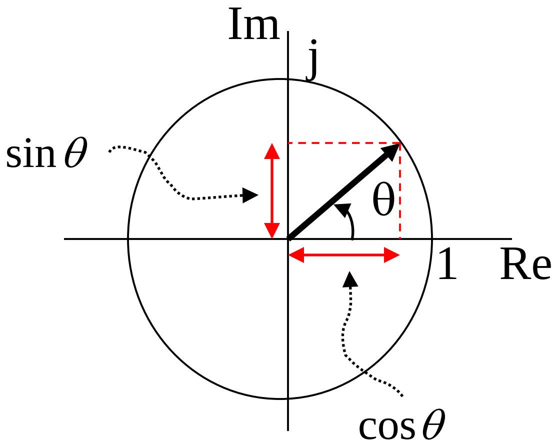
-- cos wave [e.g. cos(100t) ]

-- triangular wave

-- square wave

We only need to consider sin/cos (sinusoidal) wave in AC analysis.

## *Euler's Identity*

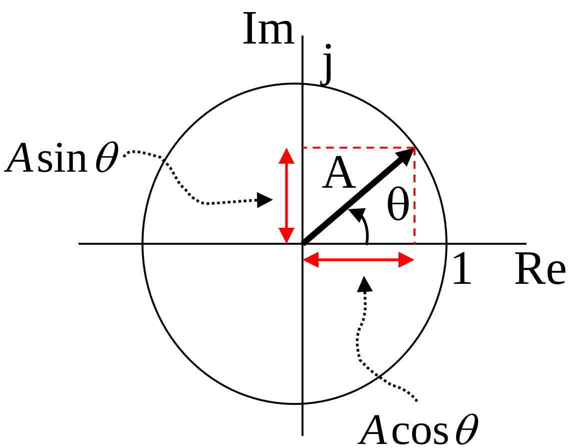



$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$\left|e^{j\theta}\right| = 1$$

$$\left|\cos\theta + j\sin\theta\right| = \sqrt{\cos^2\theta + \sin^2\theta} = 1$$

---




$$Ae^{j\theta} = A\cos\theta + jA\sin\theta = A\angle\theta$$

*Rectangular form* *Polar form*

$$\sin \omega t = \cos(\omega t - 90^\circ)$$
$$\cos \omega t = \sin(\omega t + 90^\circ)$$

---

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta \qquad \omega = 2\pi f \qquad e^{\pm j\pi} = -1$$

$$a + jb = \left(\sqrt{a^2 + b^2}\right) * e^{j\left(\tan^{-1} b/a\right)} \qquad j = \frac{1}{-j} \qquad (-j)(j) = 1$$

---

<u>*Addition (Subtraction)*</u> $\quad n_1 = 8 + j16 \qquad n_2 = 12 - j3$

$$n_1 + n_2 = (8 + 12) + j(16 - 3) = 20 + j13$$

$$n_1 - n_2 = (8 - 12) + j(16 - (-3)) = -4 + j19$$

*Addition (Subtraction)*

$$n_1 = 5\angle 18.3^\circ \qquad n_2 = 12\angle 115^\circ$$

$$\begin{aligned} n_1 + n_2 &= (4.75 + j1.57) + (-5.07 + j10.88) \\ &= -0.32 + j12.45 \\ &= 12.45\angle 91.47^\circ \end{aligned}$$

$$\begin{aligned} n_1 - n_2 &= (4.75 + j1.57) - (-5.07 + j10.88) \\ &= 9.82 - j9.31 \\ &= 13.53\angle -43.47^\circ \end{aligned}$$

*Multiplication*

$$n_1 = 4 + j5$$

$$n_2 = 2 - j3$$

$$n_1 n_2 = (4 + j5)(2 - j3)$$

$$= 8 - j12 + j10 + 15$$

$$= 23 - j2$$

$$= 23.09\angle 355°$$

---

$$n_1 = 6.4\angle 51.34°$$

$$n_2 = 3.6\angle -56.31°$$

$$n_1 n_2 = (6.4\angle 51.34°)(3.6\angle -56.31°)$$

$$= 23.04\angle 355°$$

*Division*

$$n_1 = 4 + j5$$
$$n_2 = 2 - j3$$

$$\frac{n_1}{n_2} = \frac{4 + j5}{2 - j3} = \frac{(4 + j5)(2 + j3)}{(2 - j3)(2 + j3)}$$

$$= \frac{8 + j12 + j10 - 15}{4 + 9}$$

$$= -0.54 + j1.69$$

$$= 1.78\angle 107.6°$$

---

$$n_1 = 6.4\angle 51.34°$$
$$n_2 = 3.6\angle 303.7°$$

$$\frac{n_1}{n_2} = \frac{6.4\angle 51.34°}{3.6\angle 303.7°}$$

$$= 1.78\angle -252.36°$$

$$= 1.78\angle 107.6°$$

Z = Impedance (Ω)

Y = Admittance (S) = $\dfrac{1}{Z}$

$$Z_R = R$$

$$Z_L = j\omega L$$

$$Z_C = \frac{1}{j\omega C}$$

$$V = I * R$$

$$\boldsymbol{V = I * Z}$$

**Ex.** $v_1(t) = 12\cos(180t - 3.14)$

<u>*Frequency (Hz)*</u> = **?**

**$\omega$ = 180 = 2πf**

**f = 28.6Hz**

---

**Ex.** $I = 12\angle 50°$ $R = 4\Omega$

*use* cos *function*

$frequency = 60Hz$

$v_R(t) = ?$

$V = I * Z = (12\angle 50°)(4) = 48\angle 50°$

$v(t) = A\cos(\omega t + \theta)$

$\omega = 2\pi(60) = 377\ {}^{rad}/_{s}$

$\theta = {}^{50}/_{360} * 2\pi$

$$\boxed{v(t) = 48\cos\left(377t + \frac{50}{360} * 2\pi\right)V}$$

$$\boxed{v(t) = 48\cos(377t + 50^o)V}$$

**Ex.**

$$v_c(t) = 100\cos(377t + 0.262)$$

**$i_C(t) = ?$**

$$C = 100\,\mu F$$

$$V_C = 100\angle\left(\tfrac{0.262}{2\pi} * 360\right)^\circ = 100\angle 15^\circ$$

$$Z = \frac{1}{j\omega C} = \frac{1}{j(377)(100 * 10^{-6})} = -j26.525\Omega$$

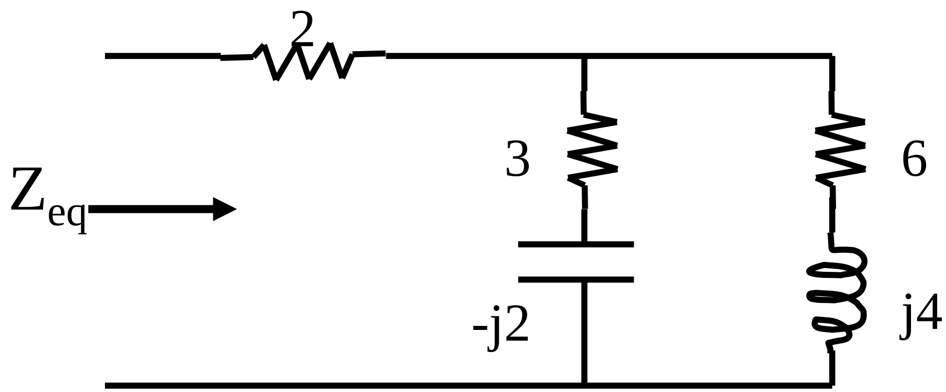
$$\boxed{V_C = I_C * Z_C}$$

$$I = \frac{V}{Z} = \frac{100\angle 15^\circ}{-j26.525} = \frac{100\angle 15^\circ}{26.525\angle -90^\circ} = 3.77\angle 105^\circ$$

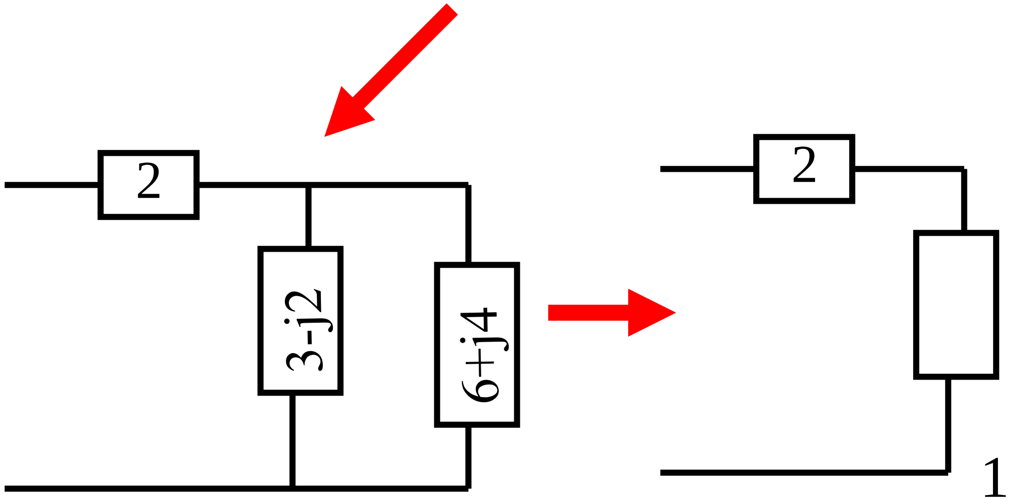
$$\boxed{v(t) = A\cos(\omega t + \theta)}$$

$$\boxed{i(t) = 3.77\cos\left(377t + \frac{105}{360} * 2\pi\right)}$$

**Ex.**




$$Z_{eq} = ?$$

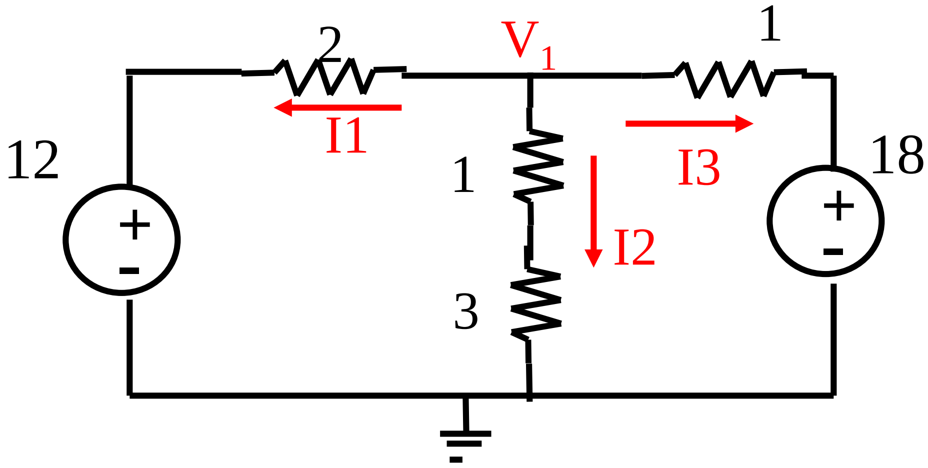



$$\frac{1}{\frac{1}{3-j2}+\frac{1}{6+j4}}$$

$$Z_{eq} = 2 + \frac{1}{\frac{1}{3-j2}+\frac{1}{6+j4}}$$

$$= 4.79\angle -7.33°$$

**Ex.**




$I_1, I_2, I_3 = ?$

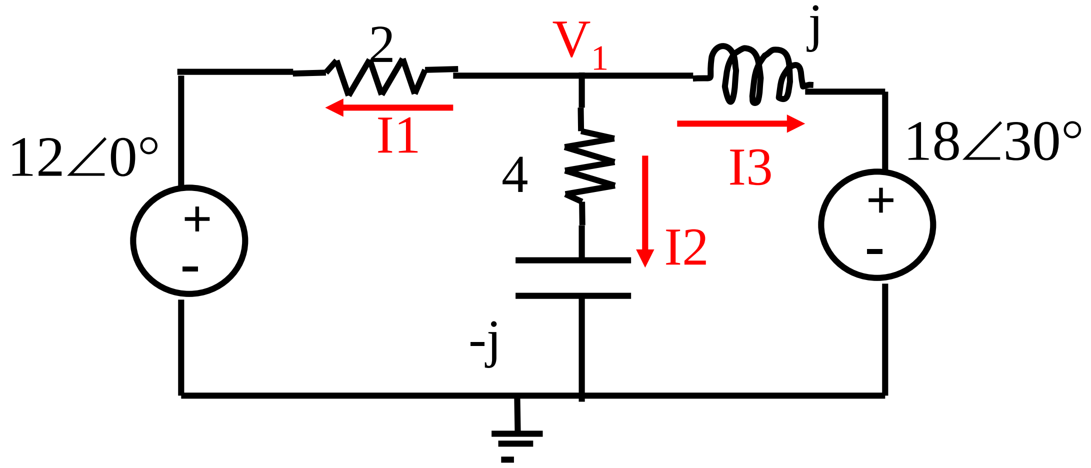
$$\frac{V_1 - 12}{2} + \frac{V_1 - 0}{4} + \frac{V_1 - 18}{1} = 0 \quad \Longrightarrow \quad V_1 = 13.714$$

$$I_1 = \frac{13.714 - 12}{2} = 0.857$$

$$I_2 = \frac{13.714 - 0}{4} = 3.429$$

$$I_3 = \frac{13.714 - 18}{1} = -4.286$$

**Ex.**




$I_1, I_2, I_3 = ?$

$$\frac{V_1 - 12\angle 0}{2} + \frac{V_1 - 0}{4 - j} + \frac{V_1 - 18\angle 30}{j} = 0 \implies V_1 = 18.11\angle 5.9°$$

$$I_1 = \frac{18.11\angle 5.9 - 12\angle 0}{2} = 3.14\angle 17.2°$$

$$I_2 = \frac{18.11\angle 5.9 - 0}{4 - j} = 4.39\angle 19.84°$$

$$I_3 = \frac{18.11\angle 5.9 - 18\angle 30}{j} = 7.54\angle -161.2°$$